

Viscosity Information from Relativistic Nuclear Collisions: How Perfect is the Fluid Observed at RHIC?

Paul Romatschke¹ and Ulrike Romatschke²

¹*Institute for Nuclear Theory, University of Washington, Box 351550, Seattle WA, 98195, USA*

²*Department of Atmospheric Sciences, University of Washington, Box 351640, Seattle WA, 98195, USA*

(Dated: February 1, 2008)

Relativistic viscous hydrodynamic fits to RHIC data on the centrality dependence of multiplicity, transverse and elliptic flow for $\sqrt{s} = 200$ GeV Au+Au collisions are presented. For standard (Glauber-type) initial conditions, while data on the integrated elliptic flow coefficient v_2 is consistent with a ratio of viscosity over entropy density up to $\eta/s \simeq 0.16$, data on minimum bias v_2 seems to favor a much smaller viscosity over entropy ratio, below the bound from the AdS/CFT conjecture. Some caveats on this result are discussed.

The success of ideal hydrodynamics for the description of heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) has led to the idea of a quark-gluon plasma behaving as a “perfect liquid”, with a very small ratio of viscosity over entropy density [1, 2, 3, 4]. An answer to the question “How perfect is the fluid observed at RHIC?” can, however, not be found using ideal hydrodynamics, but must involve a controlled quantitative understanding of non-idealities, e.g. viscous effects. If hydrodynamics can be applied to RHIC physics, then relativistic viscous hydrodynamics should be able to provide such an understanding. In particular, if one has control over the initial conditions, it should be possible to determine the size of various hydrodynamic transport coefficients, such as the shear viscosity, by a best fit of viscous hydrodynamics (VH) to experimental data. In this Letter, we aim to take a step in this direction.

For RHIC physics, since particle number in the quark-gluon plasma is ill-defined, the relevant dimensionless parameter for VH is the ratio shear viscosity η over entropy density s . Based on the correspondence between Anti-de Sitter (AdS) space and conformal field theory (CFT), it has been conjectured [5] that all relativistic quantum field theories at finite temperature and zero chemical potential have $\eta/s \geq \frac{1}{4\pi}$. To date, no physical system violating this bound has been found.

Neglecting effects from bulk viscosity and heat conductivity, the energy momentum tensor for relativistic hydrodynamics in the presence of shear viscosity is

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \Pi^{\mu\nu}. \quad (1)$$

In Eq. (1), ϵ and p denote the energy density and pressure, respectively, and u^μ is the fluid 4-velocity which obeys $g_{\mu\nu}u^\mu u^\nu = 1$ when contracted with the metric $g_{\mu\nu}$. The shear tensor $\Pi^{\mu\nu}$ is symmetric, traceless ($\Pi^\mu_\mu = 0$), and orthogonal to the fluid velocity, $u_\mu \Pi^{\mu\nu} = 0$. Conservation of the energy momentum tensor and equation of state provide five equations for the evolution of the 10 independent components of $\epsilon, p, u^\mu, \Pi^{\mu\nu}$. The remaining five equations for the evolution of $\Pi^{\mu\nu}$ are not unambiguously agreed on at present [6, 7, 8, 9, 10]. The results in

this work will be based on using the set of equations

$$\begin{aligned} (\epsilon + p)Du^\mu &= \nabla^\mu p - \Delta^\mu_\alpha d_\beta \Pi^{\alpha\beta}, \\ D\epsilon &= -(\epsilon + p)\nabla_\mu u^\mu + \frac{1}{2}\Pi^{\mu\nu}\langle\nabla_\nu u_\mu\rangle, \\ \Delta^\mu_\alpha \Delta^\nu_\beta D\Pi^{\alpha\beta} &= -\frac{\Pi^{\mu\nu}}{\tau_\Pi} + \frac{\eta}{\tau_\Pi}\langle\nabla^\mu u^\nu\rangle - 2\Pi^{\alpha(\mu}\omega^{\nu)}_\alpha \\ &\quad + \frac{1}{2}\Pi^{\mu\nu}[5D\ln T - \nabla_\alpha u^\alpha], \end{aligned} \quad (2)$$

where d_α is the covariant derivative, used to construct the time-like and space-like derivatives $D \equiv u^\alpha d_\alpha$ and $\nabla_\mu \equiv \Delta^\alpha_\mu d_\alpha$. The remaining definitions are $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, $\langle\nabla^\mu u^\nu\rangle = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$ and the vorticity $\omega_{\mu\nu} = \nabla_\nu u_\mu - \nabla_\mu u_\nu$. Both p and temperature T are related to ϵ via the QCD equation of state, for which we take the semi-realistic result from Ref. [11]. If the relaxation time τ_Π is not too small, Eq. (2) are the most general shear viscous hydrodynamic equations that are causal and guarantee that entropy can never locally decrease [12]. Formally, Eq. (2) correspond to the relativistic Navier-Stokes equations in the limit $\tau_\Pi \rightarrow 0$, but contain corrections of higher order in gradients for $\tau_\Pi > 0$.

Unfortunately, the initial conditions for a hydrodynamic description of an ultra-relativistic heavy-ion collision at RHIC are poorly known, so one has to resort to model studies. In order to describe Au+Au collisions at RHIC energies, one typically assumes the energy density along the longitudinal direction (the beam-line) to be “boost-invariant” to first approximation [13]. With this assumption, one still has to specify the energy density distribution in the plane orthogonal to the beam line (the transverse plane). At present, there exist two main classes of models for this distribution, which we will refer to as Glauber-type and Color-Glass-Condensate (CGC)-type models. In the following, only Glauber-type models will be used.

The Glauber-type models build upon the Woods-Saxon density distribution for nuclei, $\rho_A(\mathbf{x}) \sim 1/[1 + \exp((|\mathbf{x}| - R_0)/\chi)]$, where for a gold nucleus ($A=197$) we use $R_0 = 6.4$ fm/c, $\chi = 0.54$ fm/c. Integrating the Woods-Saxon distribution over the longitudinal direction (corre-

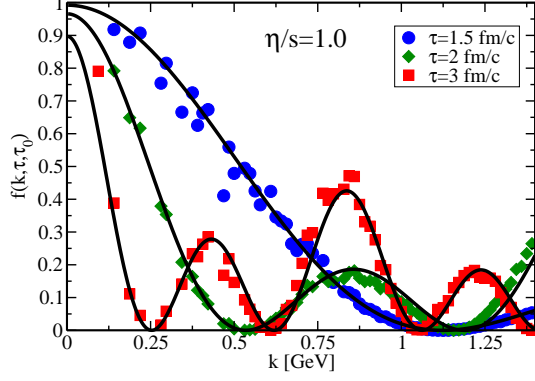


FIG. 1: Correlation function $f(k, \tau, \tau_0 = 1\text{fm/c})$ as a function of momentum k , measured for our hydrodynamics code on a 64^2 lattice with a lattice spacing of 1GeV^{-1} (symbols), compared to the “analytic” result from the linearized Eq. (2) (full lines). The good overall agreement indicates the code is solving Eq. (2) correctly in the linear regime (see [16] for details).

sponding to a Lorentz-contracted gold nucleus), one obtains the nuclear thickness function $T_A(\mathbf{x}_\perp)$. Contracting two T_A ’s for the collision of two gold nuclei at a certain impact parameter \mathbf{b} , one can define number densities in the transverse plane, such as the density of participating nuclei $n_{\text{Part}}(\mathbf{x}_\perp, \mathbf{b})$ and the density of binary collisions $n_{\text{Coll}}(\mathbf{x}_\perp, \mathbf{b})$ (see e.g. Ref. [14]). As an initial condition for hydrodynamics, one then assumes the energy density distribution ϵ in the transverse plane to be proportional to either n_{Part} or n_{Coll} or a mixture of the two. In what follows, we will concentrate on the case $\epsilon \sim n_{\text{Coll}}$, since for ideal hydrodynamics this provides a rough description of the centrality (or impact parameter) dependence of the total number of particles (“the multiplicity”) measured at RHIC [14]. Finally, for VH one also has to provide an initial condition for $\Pi^{\mu\nu}$. We choose the “minimalist assumption” $\Pi^{\mu\nu} = 0$. While one realistically expects $\Pi^{\mu\nu}$ to be nonzero initially, this assumption translates to reducing the effect of viscosity, which can serve as a baseline for future studies.

Because of boost-invariance, it is useful to work in the coordinates $\tau = \sqrt{t^2 - z^2}$ and $\eta = \text{arctanh}(z/t)$ rather than t, z . In these coordinates, boost-invariance dictates $u^\eta = 0$, so because of $u_\mu u^\mu = 1$, the only non-trivial fluid velocities can be chosen as u^x, u^y , which are assumed to vanish initially.

Before discussing results from the numerics, one can get some intuition of viscous effects on experimental observables by imagining the system to have a friction force proportional to velocity. In a heavy-ion collision, the expansion (at least initially) is strongest along the beam axis, therefore one expects viscosity to counteract this expansion. In τ, η coordinates this is achieved by a reduction of the effective longitudinal pressure $p - \Pi_\eta^\eta$ through

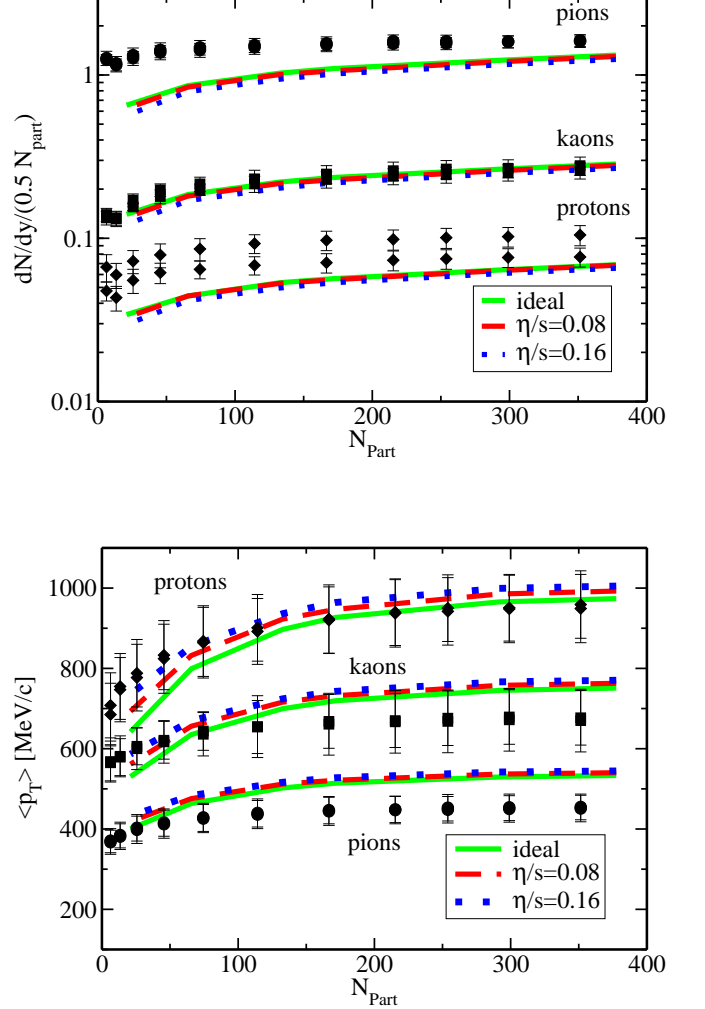


FIG. 2: Total multiplicity dN/dy and mean momentum for $\pi^+, \pi^-, K^+, K^-, p$ and \bar{p} from PHENIX [23] for Au+Au collisions at $\sqrt{s} = 200$ GeV, compared to our hydrodynamic model for various viscosity ratios η/s .

$\Pi_\eta^\eta > 0$. Since initially $\Pi_\tau^\tau \ll \Pi_\eta^\eta$ but $\Pi_\mu^\mu = 0$, the difference between equilibrium pressure p and effective longitudinal pressure has to appear as excess pressure in the transverse plane. Therefore, viscosity should lead to higher transverse velocities (“radial flow”) as compared to ideal hydrodynamics, which is indeed the case [15, 16]. Similarly, one can get an intuition of viscosity on elliptic flow v_2 (the main angular modulation of radial flow for non-central collisions): having a stronger reduction effect on higher velocities, viscosity tends to decrease velocity differences and hence elliptic flow. This agrees with the qualitative trend found by Teaney [17].

To solve Eq. (2) and treat the freeze-out (see below), we have used a two-dimensional generalization of the algorithm outlined in Ref. [16]. Details of the calculation will be given elsewhere [18]. We have checked that our

algorithm agrees with the results from Ref. [19] for central collisions, when dropping the extra terms in Eq. (2). Also, our code passes the fluctuation test from Ref. [16], shown in Fig. 1. We thus have some confidence that our numerical algorithm solves Eq. (2) correctly.

When solving the set of equations (2), we set the ratio η/s to be constant throughout the evolution of the system, since modeling any space-time dependence would necessarily introduce more unknown parameters. Therefore, results on η/s quoted below should be considered as mean values over the entire system evolution.

To make contact with experiment, the hydrodynamic variables are translated into particle spectra via the Cooper-Frye freeze-out mechanism [20] (adapted to VH [8, 16], see also [17]). For simplicity, we use a single freeze-out temperature T_f but include the effect of resonance decays with masses up to 2 GeV on the spectra [21, 22]. The normalization of the initial energy density and T_f are chosen such that the experimental data on total multiplicity and mean transverse momentum $\langle p_T \rangle$ as a function of total number of participants $N_{\text{Part}} = \int d^2\mathbf{x}_\perp n_{\text{Part}}(\mathbf{x}_\perp, \mathbf{b})$ are reasonably reproduced by our model (see Fig. 2). We choose to fit to kaons rather than pions because the former are influenced less by Bose enhancement effects, which we have ignored [19]. Note that for simplicity our model does not include a finite baryon chemical potential, prohibiting us to distinguish particles from anti-particles. As a consequence, results for protons cannot be expected to match experimental data. Starting from ideal hydrodynamics with a freeze-out temperature $T_f = 150$ MeV, we have found that reasonable fits to dN/dy and $\langle p_T \rangle$ for VH can be accomplished by keeping T_f fixed and reducing the initial entropy density by 75 η/s percent to correct for the viscous entropy production [19].

In Fig. 3 we compare our hydrodynamic model with the above fit parameters to experimental data on the integrated and minimum bias elliptic flow v_2 , respectively. Shown are results for ideal hydrodynamics and VH for the initial condition $\epsilon \sim n_{\text{Coll}}$ at an initial time $\tau_0 = 1$ fm/c. The results hardly change when assuming instead $s \sim n_{\text{Part}}$ as initial condition (see also [14]) or varying τ_0 by a factor of two. Interestingly, we also find that changing τ_Π hardly affects the results shown. Note that this depends on the presence of the terms in the last line of Eq. (2): if these terms are dropped, increasing τ_Π tends to further suppress v_2 in line with the trend found in [19].

For the above initial conditions, we have noted that there is also hardly any effect from the vorticity term. This can be understood as follows: noting that for $u^\eta = 0$ the only non-trivial vorticity is ω^{xy} , which vanishes initially because of $u^x = u^y = 0$ and forming the combination $\nabla^x Du^y - \nabla^y Du^x$ we find –up to third order

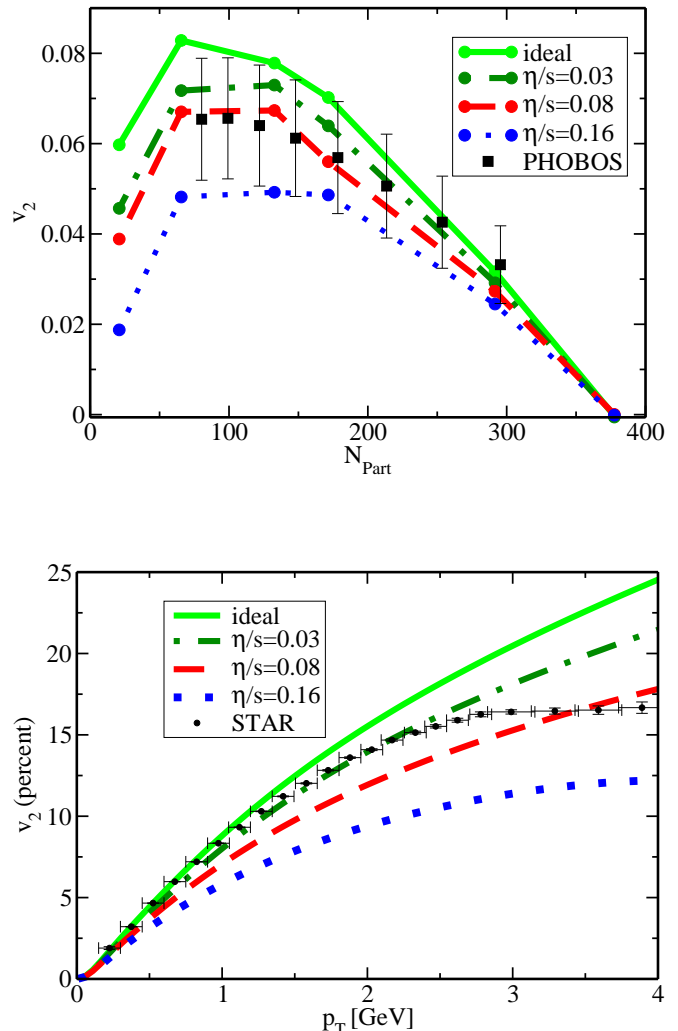


FIG. 3: PHOBOS [24] data on p_T integrated v_2 and STAR [25] data on minimum bias v_2 , for charged particles in Au+Au collisions at $\sqrt{s} = 200$ GeV, compared to our hydrodynamic model for various viscosity ratios η/s . Error bars for PHOBOS data show 90% confidence level systematic errors while for STAR only statistical errors are shown.

corrections–

$$D\omega^{xy} + \omega^{xy} \left[\nabla_\mu u^\mu + \frac{Dp}{\epsilon + p} - \frac{Du^\tau}{u^\tau} \right] = \mathcal{O}(\Pi^3). \quad (3)$$

This is the relativistic generalization of the vorticity equation, well known in atmospheric sciences [26]. Starting from $\omega^{xy} = 0$, Eq. (3) implies a very slow buildup of vorticity, explaining the tiny overall effect of the vorticity term in Eq. (2). Note that upon dropping the assumption $u^\eta = 0$, this term can become important [27].

From Fig. 3 it can be seen that the effect from viscosity on the elliptic flow is strong, in line with estimates from Ref. [17]. Data on integrated v_2 is fairly well reproduced by a viscosity of $\eta/s \sim 0.08$ and – within systematic errors – seems to be consistent with $\eta/s \sim 0.16$.

These values agree with recent estimates by other groups [28, 29, 30] and a lattice QCD calculation [31]. However, the comparison to data for minimum bias v_2 in Fig. 3 suggests that the ratio of η/s is actually smaller than the conjectured minimal bound $\eta/s = \frac{1}{4\pi} \simeq 0.08$. As mentioned, this seems to be independent from whether one adopts $\tau_{\Pi} = 6 \eta/(\epsilon + p)$, the weak-coupling QCD result, or extrapolates to $\tau_{\Pi} \rightarrow 0$, which is very close to the AdS/CFT value found in [32]. Indeed, the minimum bias v_2 seems to favor $\eta/s \simeq 0.03$, at least at low momenta, where hydrodynamics is supposed to be most applicable. Note that this result could change drastically if the minimum bias data were decreased by 20%, which is the estimated systematic error quoted in [25].

There are, however, a number of caveats that should be considered before taking the above numbers literally. Firstly, we have only considered Glauber-type initial conditions, and assumed $\Pi^{\mu\nu}(\tau_0) = 0$. It has been suggested that CGC-type initial conditions lead to larger overall v_2 [33] which in turn would raise the allowed values for η/s in our calculation. This is due to the larger eccentricities in this model [34] (note the issues raised in [35]). However, larger eccentricities in general also lead to a faster build-up of transverse flow, which is further enhanced by viscosity. Thus, when required to fit all the data in Figs. 2 and 3, it is unclear whether this CGC-type model will predict substantially higher η/s than found here.

Secondly, we used VH until the last scattering instead of more sophisticated hydro+cascade models (e.g. [36, 37]). We do expect changes in the extracted values of η/s once a VH+cascade model description becomes available. Finally, at present we cannot exclude that effects not captured by hydrodynamics, such as strong mean-fields, distort our results. Work on QCD plasma instabilities and CGC dynamics might shed some light on this issue.

To summarize, we have presented the first viscous hydrodynamic fits to experimental data on the centrality dependence of dN/dy , $\langle p_T \rangle$ and v_2 at top RHIC energies. For Glauber-type initial conditions, we found that data seems to favor values for η/s which are very small, below the AdS/CFT bound [41]. While suggested to be possible in [39, 40], it will be interesting to see whether the above caveats – once addressed – can change our results enough to accommodate viscosity equal or larger than the bound. In any case, we hope that our work can serve as a guideline to understanding the properties of the fluid created at RHIC.

PR would like to thank P. Huovinen and T. Lappi for fruitful discussions. This work was partially supported by the US Department of Energy, grant number DE-FG02-00ER41132.

-
- [1] D. Teaney, J. Lauret and E. V. Shuryak, Phys. Rev. Lett. **86** (2001) 4783.
 - [2] P. Huovinen *et al.*, Phys. Lett. B **503** (2001) 58.
 - [3] T. Hirano and K. Tsuda, Phys. Rev. C **66**, 054905 (2002).
 - [4] M. J. Tannenbaum, Rept. Prog. Phys. **69** (2006) 2005.
 - [5] P. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. **94** (2005) 111601.
 - [6] A. Muronga, Phys. Rev. C **69** (2004) 034903.
 - [7] U. W. Heinz, H. Song and A. K. Chaudhuri, Phys. Rev. C **73** (2006) 034904.
 - [8] R. Baier, P. Romatschke and U. A. Wiedemann, Phys. Rev. C **73** (2006) 064903.
 - [9] K. Tsumura, T. Kunihiro and K. Ohnishi, arXiv:hep-ph/0609056.
 - [10] T. Koide, G. S. Denicol, Ph. Mota and T. Kodama, Phys. Rev. C **75** (2007) 034909.
 - [11] M. Laine and Y. Schroder, Phys. Rev. D **73** (2006) 085009.
 - [12] W. Israel, Ann.Phys. **100** (1976) 310; W. Israel and J.M. Stewart, Phys. Lett. **58A** (1976) 213; Ann.Phys. **118**, (1979) 341; I-Shih Liu, I. Müller and T. Ruggeri, Ann.Phys. **169** (1986) 191. W.A. Hiscock and L. Lindblom, Phys. Rev. D **35**, 3723 (1987).
 - [13] J. D. Bjorken, Phys. Rev. D **27** (1983) 140.
 - [14] P. F. Kolb *et al.*, Nucl. Phys. A **696** (2001) 197.
 - [15] A. K. Chaudhuri and U. W. Heinz, J. Phys. Conf. Ser. **50**, 251 (2006).
 - [16] R. Baier and P. Romatschke, arXiv:nucl-th/0610108.
 - [17] D. Teaney, Phys. Rev. C **68** (2003) 034913.
 - [18] P. Romatschke and U. Romatschke, in preparation.
 - [19] P. Romatschke, arXiv:nucl-th/0701032.
 - [20] F. Cooper and G. Frye, Phys. Rev. D **10**, 186 (1974).
 - [21] J. Sollfrank, P. Koch and U. W. Heinz, Phys. Lett. B **252** (1990) 256.
 - [22] J. Sollfrank, P. Koch and U. W. Heinz, Z. Phys. C **52** (1991) 593.
 - [23] S. S. Adler *et al.* [PHENIX Collaboration], Phys. Rev. C **69** (2004) 034909.
 - [24] B. Alver *et al.* [PHOBOS Collaboration], arXiv:nucl-ex/0702036.
 - [25] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **92** (2004) 062301.
 - [26] James R. Holton, “An Introduction to Dynamic Meteorology”, International geophysics series, 88, 4th edition, Elsevier Academic Press, Amsterdam (2004).
 - [27] V. Rezania and R. Maartens, Phys. Rev. Lett. **84** (2000) 2560.
 - [28] S. Gavin and M. Abdel-Aziz, Phys. Rev. Lett. **97** (2006) 162302.
 - [29] R. A. Lacey *et al.*, Phys. Rev. Lett. **98** (2007) 092301.
 - [30] H. J. Drescher, A. Dumitru, C. Gombeaud and J. Y. Ollitrault, arXiv:0704.3553 [nucl-th].
 - [31] H. B. Meyer, arXiv:0704.1801 [hep-lat].
 - [32] M. P. Heller and R. A. Janik, Phys. Rev. D **76** (2007) 025027.
 - [33] T. Hirano *et al.*, Phys. Lett. B **636** (2006) 299.
 - [34] H. J. Drescher, A. Dumitru, A. Hayashigaki and Y. Nara, Phys. Rev. C **74** (2006) 044905.
 - [35] T. Lappi and R. Venugopalan, Phys. Rev. C **74** (2006) 054905.
 - [36] S. A. Bass and A. Dumitru, Phys. Rev. C **61** (2000)

- 064909.
- [37] T. Hirano and M. Gyulassy, Nucl. Phys. A **769** (2006) 71.
 - [38] U.W. Heinz, talk given at “Last Call for LHC Predictions”, CERN, May 2007.
 - [39] T. D. Cohen, Phys. Rev. Lett. **99** (2007) 021602.
 - [40] M. Lublinsky and E. Shuryak, Phys. Rev. C **76** (2007) 021901.
 - [41] While this work was being finalized, we became aware of similar findings by another group [38].